



## **Initial right nano topology via directed graph for human lung modeling**

**M. Dhanapal<sup>1\*</sup> and K. Rekha<sup>2</sup>**

<sup>\*1</sup>Assistant Professor, Department of Mathematics,  
Kongunadu College of Engineering and Technology (Autonomous), Tiruchirappalli-621215.  
Research Scholar (Part Time), PG and Research Department of Mathematics,  
Bishop Heber College (Autonomous), Affiliated to Bharathidasan University,  
Tiruchirappalli-620017, Tamilnadu, India.

<sup>2</sup> Assistant Professor, PG and Research Department of Mathematics,  
Bishop Heber College (Autonomous), Affiliated to Bharathidasan University,  
Tiruchirappalli-620017, Tamilnadu, India.

<sup>1\*</sup>Corresponding Author Mail Id: [mdhanapal46@gmail.com](mailto:mdhanapal46@gmail.com)

### **ABSTRACT:**

This research presents a foundational right nano topology using an initial right neighborhood structure. This framework is created based on the directed graph of a human lung model system.

**Keywords:** Directed graph, initial right neighborhoods, initial right lower approximation, initial right upper approximation and initial right boundary region.

### **1. INTRODUCTION**

Nano topology has emerged as an significant area of research for study discrete and approximate structure, mainly when combined with graph theory concepts. In latest model, several researchers have explored the development of nano-topology using neighborhood systems from directed graphs. For example, Nawar and El-Atik [1] introduced a directed graph based nano topological model of the human heart, representing its effectiveness in medical analysis. Similarly, Lellis Thivagar et al. [2] investigated the structural aspects of nano topology through the graph theory approaches. Graph theory has also been efficiently applied in modeling biological systems. Abeyrathne and Lanel [3] developed a directed graph model for the human blood circulatory system, enabling the analysis of relationships between anatomical components. In addition, Parimala et al., [4] explored structural isomorphism using nano-topological techniques, emphasizing the role of vertex neighborhoods in deriving approximations and induced topologies. The relationship between directed graphs and biological processes was further examined by Hakeem et al. [5], who studied pathless directed topology in connection with blood circulation in the human heart. Further advancements consist of the work of Abu-Gdairi et al. [6], who applied topological visualization techniques to graph theory model representations of the human heart. In previous method, Rekha and Dhanapal [7] developed nano topology using graph models of the human double circulation system, while Ibrahim and Khalaf [8] introduced nano topology induced by directed graphs and analyzed their closure and interior properties. This approach by Dhanapal M and Rekha K provides a novel framework for analyzing complex biological systems and extends the application of nano-topology in graph-based modeling, particularly in the context of human lung analysis [9].



Motivated by these contributions, the present study focuses on constructing initial right nano-topological structures using initial right neighborhood systems derived from directed graphs. In particular, a directed graph model of the human lung is considered, and initial right neighborhoods are defined based on initial right lower and initial right upper approximations as well as initial right boundary regions. Using these structures, an initial right nano-topology is developed over selected subgraphs.

## 2. Preliminaries

### Definition:2.1

The equivalence relation defined on the set  $(U)$  is represented by  $(R)$  and  $U$  be the universe and  $\tau_R(E) = \{U, \varphi, L_R(E), U_R(E), B_R(E)\}$  where  $E \subseteq U$ .  $\tau_R(E)$  fulfils the axioms:

- (i)  $U$  and  $\varphi \in \tau_R(E)$ .
- (ii)  $\tau_R(E)$  includes the union of all components belonging to any chosen subcollection.
- (iii)  $\tau_R(E)$  includes the intersection of all components within any finite subcollection.

On the set  $U$ ,  $\tau_R(E)$  generates a nano-topology corresponding to  $E$ . The nano-topological space is represented as  $\{U, \tau_R(E)\}$ , where the term nano-topology refers to the structural properties defined by  $\tau_R(E)$ .

### Definition: 2.2

If a graph doesn't contain loops or connections that relate the same pair of vertices twice, it is considered as simple graph. A null graph is a graph with no edges.

### Definition: 2.3

A graph  $G(\rho, \sigma)$  consists of a nonempty set of vertices  $\rho = \rho(G)$  and a set of edges  $\sigma = \sigma(G)$ , which is a subset of unordered pairs of elements from  $\rho$ . If  $G$  is finite (or infinite), then  $\rho(G)$  likewise finite (or infinite). The degree of a vertex  $u$  in  $\rho(G)$  is defined as the number of edges incident to it, a vertex with degree zero is termed an *isolated vertex*. An edge connecting a vertex to itself is known as a *loop*, while an edge connecting two distinct vertices is referred to as a link.

### Description: 2.4

If  $u, o \in O$  and  $G(\rho, \sigma)$  is a directed graph, then:

\* In case  $\overline{u\rho} \in \sigma(G)$ , let  $u$  be the left vertex .

\* If  $\overline{\rho u} \in \sigma(G)$ , then  $u$  be a right vertex .

\* The count of vertices  $u$  for which  $\overline{u\rho} \in \sigma(G)$  is termed the *left degree* of the vertex  $\rho$ .

\* The number of vertices  $u$  satisfying  $\overline{\rho u} \in \sigma(G)$  is called the *right degree* of the vertex  $\rho$ .

### Definition: 2.5

If  $\hat{G}(\hat{\rho}, \hat{\sigma})$  represents a digraph and  $\hat{\rho} \in \hat{\theta}(\hat{G})$ , then the new vertices corresponding to the initial neighborhoods are determined as follows:

(1) Initial left neighbourhoods:  $\hat{N}_l^i(\hat{\rho}) = \{\hat{u} \in \hat{\theta}(\hat{G}) / \hat{N}_l(\hat{\rho}) \subseteq \hat{N}_l(\hat{u})\}$

(2) Initial right neighbourhoods:  $\hat{N}_r^i(\hat{\rho}) = \{\hat{u} \in \hat{\theta}(\hat{G}) / \hat{N}_r(\hat{\rho}) \subseteq \hat{N}_r(\hat{u})\}$

(3) Union of initial neighbourhoods:  $\hat{N}_u^i(\hat{\rho}) = \hat{N}_l^i(\hat{\rho}) \cup \hat{N}_r^i(\hat{\rho})$

(4) Intersection of initial neighbourhoods:  $\hat{N}_i^i(\hat{\rho}) = \hat{N}_l^i(\hat{\rho}) \cap \hat{N}_r^i(\hat{\rho})$ .

### Definition: 2.6

Suppose  $\hat{L} \subseteq \hat{G}$  is a subgraph,  $N_r^i(\hat{\rho})$  corresponds to the initial right neighborhoods of  $\hat{\rho} \in$



$\hat{\theta}(\hat{G})$  then,

(i) Initial right lower-approximation is  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{\hat{\rho} / N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L})\}$

(ii) Initial right upper-approximation is  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{\hat{\rho} / N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \varphi\}$

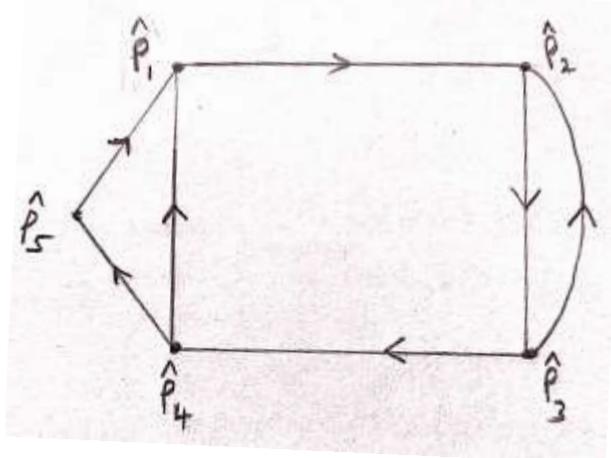
(iii) Initial right boundary region is  $\hat{B}_{N_r^i}[\hat{\theta}(\hat{L})] = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$

That is  $\hat{\tau}_{N_r^i}[\hat{\theta}(\hat{L})] = \{\hat{\theta}(\hat{G}), \varphi, \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})], \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})], \hat{B}_{N_r^i}[\hat{\theta}(\hat{L})]\}$  describes the initial right nanotopology of a graph  $\hat{G}(\hat{\theta}, \hat{\sigma})$  captioned "Graph's tempting initial right nanotopology" where each corresponds to a vertex  $N_r^i(\hat{\rho})$ .

We call  $(\hat{\theta}(\hat{G}), \hat{\tau}_{N_r^i}[\hat{\theta}(\hat{L})])$  as nano-topological space stimulated by initial right neighbourhood.

**Example: 2.7**

Contemplate a directed graph  $\hat{G}$  with  $\hat{\theta}(\hat{G}) = \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\rho}_5\}$  and  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{G})$



**Figure 2.1:** Directed graph  $\hat{G}$

Next, we obtain,

**Table: 2.1** Initial right neighbourhoods.

Right Neighbourhood	Initial right neighbourhood
$N_r(\hat{\rho}_1) = \{\hat{\rho}_1, \hat{\rho}_2\}$	$N_r^i(\hat{\rho}_1) = \{\hat{\rho}_1\}$
$N_r(\hat{\rho}_2) = \{\hat{\rho}_2, \hat{\rho}_3\}$	$N_r^i(\hat{\rho}_2) = \{\hat{\rho}_2, \hat{\rho}_3\}$
$N_r(\hat{\rho}_3) = \{\hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4\}$	$N_r^i(\hat{\rho}_3) = \{\hat{\rho}_3\}$
$N_r(\hat{\rho}_4) = \{\hat{\rho}_1, \hat{\rho}_4, \hat{\rho}_5\}$	$N_r^i(\hat{\rho}_4) = \{\hat{\rho}_4\}$
$N_r(\hat{\rho}_5) = \{\hat{\rho}_1, \hat{\rho}_5\}$	$N_r^i(\hat{\rho}_5) = \{\hat{\rho}_5\}$

**Table: 2.2** Initial right Nano topologies for probable sub-graphs of G through the use of initial right neighbourhoods.

$\hat{\theta}(\hat{L})$	$\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$	$\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$	$\hat{B}_{N_r^i}[\hat{\theta}(\hat{L})]$	$\hat{\tau}_{N_r^i}[\hat{\theta}(\hat{L})]$
$\{\hat{\rho}_1\}$	$\{\hat{\rho}_1\}$	$\{\hat{\rho}_1\}$	$\varphi$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}\}$
$\{\hat{\rho}_2\}$	$\varphi$	$\{\hat{\rho}_2\}$	$\{\hat{\rho}_2\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_2\}\}$





$\varphi$	$\varphi$	$\varphi$	$\varphi$	$\{\hat{\theta}(\hat{G}), \varphi\}$
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**3. Theorems on intial right neighbourhood**

**Theorem: 3.1**

If  $\hat{G}(\hat{\theta}, \hat{\sigma})$  is a digraph and  $N_r^i(\hat{\rho})$  be denoted by an initial right neighbourhood of  $\hat{\rho} \in \hat{\theta}(\hat{G})$  with any two sub graphs  $\hat{L}$  and  $\hat{T}$  of  $\hat{G}$ , then the consequent statements are holds:

- (i)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \hat{\theta}(\hat{L}) \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$
- (ii)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \hat{\theta}(\hat{L})$
- (iii)  $\underline{L}_{N_r^i}[\varphi] = \overline{L}_{N_r^i}[\varphi] = \varphi$
- (iv) If  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{T})$  then  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$  and  $\overline{U}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \overline{U}_{N_r^i}[\hat{\theta}(\hat{T})]$
- (v)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \left[ \overline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c] \right]^c$
- (vi)  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \left[ \underline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c] \right]^c$

**Proof:** Proof of (i): Let  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$

By definition of  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$ ,

$$N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L})$$

$$\Rightarrow \hat{\rho} \in \hat{\theta}(\hat{L}),$$

$$\Rightarrow \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \hat{\theta}(\hat{L}) \text{ -----(i)}$$

By definition of  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$ ,

$$N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset$$

$$\Rightarrow \hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] ,$$

$$\Rightarrow \hat{\theta}(\hat{L}) \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \text{ -----(ii)}$$

From (i) and (ii), we get,

$$\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \hat{\theta}(\hat{L}) \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$$

Proof of (ii): Let  $\hat{\rho} \in \hat{\theta}(\hat{L})$

Then  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \hat{\theta}(\hat{L})$

$$\Rightarrow \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \hat{\theta}(\hat{L})$$

Thus  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$ ,

Hence  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \hat{\theta}(\hat{L}) \text{ -----(a)}$

Let  $\hat{\rho} \in \hat{\theta}(\hat{L})$ ,  $N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L})$

$$N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset$$

Thus  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$

Hence  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \hat{\theta}(\hat{L}) \text{ -----(b)}$

From (a) and (b), we get

$$\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \hat{\theta}(\hat{L})$$

Proof of (iii): Let  $\emptyset \in \hat{\theta}(\hat{L})$

Then  $N_r^i[\emptyset] = \emptyset$

But  $N_r^i[\emptyset] \subseteq \emptyset$



$$\Rightarrow \underline{L}_{N_r^i}[\emptyset] = \emptyset \text{-----(a)}$$

But  $N_r^i(\hat{\rho}) \cap \emptyset = \emptyset$

$$\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \emptyset \text{-----(b)}$$

From (a) and (b), we get

$$\underline{L}_{N_r^i}[\emptyset] = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \emptyset$$

Proof of (iv):

Let  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \text{-----(a)}$

$$\Rightarrow N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L}) \text{ and } \rho \in \hat{\theta}(\hat{L}),$$

We know that  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{T})$ ,

$$\text{i.e., } N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{T})$$

Hence  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \text{-----(b)}$

From (a) and (b) we have,  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$

Let  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \text{-----(c)}$

$$\Rightarrow N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset$$

$$\text{i.e., } N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{T})$$

$$\therefore N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset$$

Hence,  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \text{-----(d)}$

From (c) and (d) we have,

$$\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})].$$

Proof of (v):

Let  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset \}$

i.e.,  $\overline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap (\hat{\theta}(\hat{L}))^c \neq \emptyset \}$

$$[\overline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c]]^c = \left\{ \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap (\hat{\theta}(\hat{L}))^c \neq \emptyset \} \right\}^c$$

$$= \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \left\{ \frac{\hat{\rho}}{N_r^i(\hat{\rho})} \cap (\hat{\theta}(\hat{L}))^c = \emptyset \right\}$$

$$= \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{S}) \neq \emptyset \}$$

$$\text{i.e., } [\overline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c]]^c = \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})].$$

Proof of (vi):

Let  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L}) \}$

i.e.,  $\underline{L}_{N_r^i}[(\hat{\theta}(\hat{L}))^c] = \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \subseteq (\hat{\theta}(\hat{S}))^c \}$

$$= \left\{ \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \subseteq (\hat{\rho}(\hat{S}))^c \} \right\}^c$$

$$= \left\{ \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \subseteq \hat{\theta}(\hat{L}) \} \right\}^c$$

$$= \left\{ \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap \theta(\hat{L}) = \emptyset \} \right\}^c$$

$$= \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \emptyset \} = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})].$$



**Theorem: 3.2**

Let  $\hat{G}(\hat{\theta}, \hat{\rho})$  be a directed graph,  $N_r^i(\hat{\rho})$  being an initial right neighbourhood with any two sub graphs  $\hat{L}$  and  $\hat{T}$  of  $\hat{G}$ . Then the following statements are holds:

- (i)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$
- (ii)  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$
- (iii)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] = \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$
- (iv)  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})] = \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$

**Proof:**

Proof of (i): Since,  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$  and  $\hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$ ,  
 Then  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$  and  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$   
 $\Rightarrow \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$ .

Proof of (ii): Since,  $\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{L})$  and  $\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{T})$ ,  
 Then  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  and  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ .  
 $\therefore \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ .

Proof of (iii): Since,  $\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{K}) \subseteq \hat{\theta}(\hat{L})$  and  $\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{T})$ ,  
 Then  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  and  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ .  
 $\therefore \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{K})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ ------(1)

Let  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ ------(a)

Then  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  and  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$

By definition of  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  we have,  $N_r^i(\hat{\rho}) \subseteq [\hat{\theta}(\hat{L})]$  and  $N_r^i(\hat{\rho}) \subseteq [\hat{\theta}(\hat{T})]$

Then  $\hat{\rho} \in \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})]$ ------(b)

From statements (a) and (b), we obtain,

$$\underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})]$$
------(2)

Hence by (1) and (2),  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap \hat{\theta}(\hat{T})] = \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$ .

Proof of (iv): Since,  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$  and  $\hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$ ,  
 Then  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$  and  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$

Since,  $\hat{\theta}(\hat{L}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$  and  $\hat{\theta}(\hat{T}) \subseteq \hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})$ ,  
 Then  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$  and  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$   
 $\Rightarrow \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$ ------(1)

Let  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})]$ ------(a)

Then  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  and  $\hat{\rho} \in \overline{L}_{N_r^i}[\hat{\theta}(\hat{T})]$

By definition of  $\overline{L}_{N_r^i}[\hat{\theta}(\hat{L})]$  we have,

$$\begin{aligned} \overline{L}_{N_r^i}[\hat{\theta}(\hat{L})] &= \cup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L}) \neq \varphi \} \\ &\Rightarrow \hat{\rho} \in \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / N_r^i(\hat{\rho}) \cap [\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})] \neq \varphi \} \end{aligned}$$



$$\Rightarrow \hat{\rho} \in \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / [N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L})] \cup [N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{T})] \neq \varphi \}$$

$$\Rightarrow \hat{\rho} \in \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / [N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{L})] \neq \varphi \} \text{ and } \hat{\rho} \in \bigcup_{\hat{\rho} \in \hat{\theta}(\hat{G})} \{ \hat{\rho} / [N_r^i(\hat{\rho}) \cap \hat{\theta}(\hat{T})] \neq \varphi \}$$

$$\Rightarrow \rho \in \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] \text{ and } \hat{\rho} \in \bar{L}_{N_r^i}[\theta(\hat{T})]$$

$$\text{i.e., } \hat{\rho} \in \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \bar{L}_{N_r^i}[\theta(\hat{T})] \quad \text{----- (b)}$$

Equations (a) and (b) give,  $\bar{L}_{N_r^i}[\theta(\hat{L}) \cup \hat{\theta}(\hat{T})] \subseteq \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cup \bar{L}_{N_r^i}[\theta(\hat{T})]$  ----- (2)

Hence by (1) and (2),

$$\bar{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cup \hat{\theta}(\hat{T})] = \bar{L}_{N_r^i}[\theta(\hat{L})] \cup \bar{L}_{N_r^i}[\hat{\theta}(\hat{T})].$$

**Theorem: 3.3**

Let  $\hat{G}(\hat{\theta}, \hat{\sigma})$  be a directed graph,  $N_r^i(\hat{\rho})$  being an initial right neighbourhood with any two sub graphs  $\hat{L}$  and  $\hat{T}$  of  $\hat{G}$ . Then the following statements are holds:

- (i)  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) - \theta(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \underline{L}_{N_r^i}[\theta(\hat{T})]$
- (ii)  $\bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \bar{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \bar{L}_{N_r^i}[\hat{\theta}(\hat{L}) - \hat{\theta}(\hat{T})]$

**Proof:**

Proof of (i):

Let  $\hat{\theta}(\hat{L}) - \hat{\theta}(\hat{T}) = \hat{\theta}(\hat{L}) \cap [\hat{\theta}(\hat{G}) - \hat{\theta}(\hat{T})]$ ,

Then  $\underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) - \hat{\theta}(\hat{T})] = \underline{L}_{N_r^i}[\hat{\theta}(\hat{L}) \cap [\theta(\hat{G}) - \hat{\theta}(\hat{T})]]$

$$= \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \underline{L}_{N_r^i}[\hat{\theta}(\hat{G}) - \hat{\theta}(\hat{T})]$$

$$= \underline{L}_{N_r^i}[\theta(\hat{L})] \cap [\hat{\theta}(\hat{G}) - \underline{L}_{N_r^i}\hat{\theta}(\hat{T})]$$

i.e.,  $\underline{L}_{N_r^i}[\theta(\hat{L}) - \hat{\theta}(\hat{T})] \subseteq \underline{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \underline{L}_{N_r^i}\hat{\theta}(\hat{T})]$

Proof of (ii):

Let  $\hat{\theta}(\hat{L}) - \hat{\theta}(\hat{T}) = \theta(\hat{L}) \cap [\hat{\theta}(\hat{G}) - \hat{\theta}(\hat{T})]$ ,

Then  $\bar{L}_{N_r^i}[\hat{\theta}(\hat{L}) - \theta(\hat{T})] = \bar{L}_{N_r^i}[\theta(\hat{L}) \cap [\hat{\theta}(\hat{G}) - \hat{\theta}(\hat{T})]]$

$$= \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap \bar{L}_{N_r^i}[\hat{\theta}(\hat{G}) - \hat{\theta}(\hat{T})]$$

$$= \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] \cap [\hat{\theta}(\hat{G}) - \bar{L}_{N_r^i}\hat{\theta}(\hat{T})]$$

$$\subseteq \bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \bar{L}_{N_r^i}[\hat{\theta}(\hat{T})]$$

i.e.,  $\bar{L}_{N_r^i}[\hat{\theta}(\hat{L})] - \bar{L}_{N_r^i}[\hat{\theta}(\hat{T})] \subseteq \bar{L}_{N_r^i}[\hat{\theta}(\hat{L}) - \hat{\theta}(\hat{T})]$ .

**4. INITIAL RIGHT NANO-TOPOLOGICAL SPACES VIA GRAPHS FOR HUMAN LUNG MODELING**

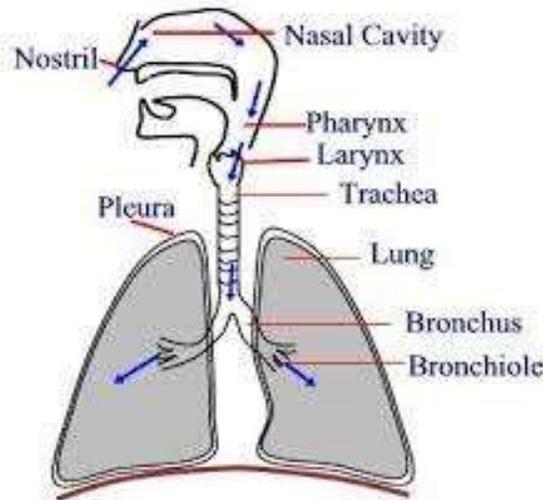
The human lung is a significant piece of the human respiratory framework. Lungs go about as the primary organ for taking in oxygen and removing carbon-dioxide, which gives energy to active work.

Here we shows that different parts of the lung in Figure-4.1 and the corresponding graphical representation of lung functioning in Figure-4.2.

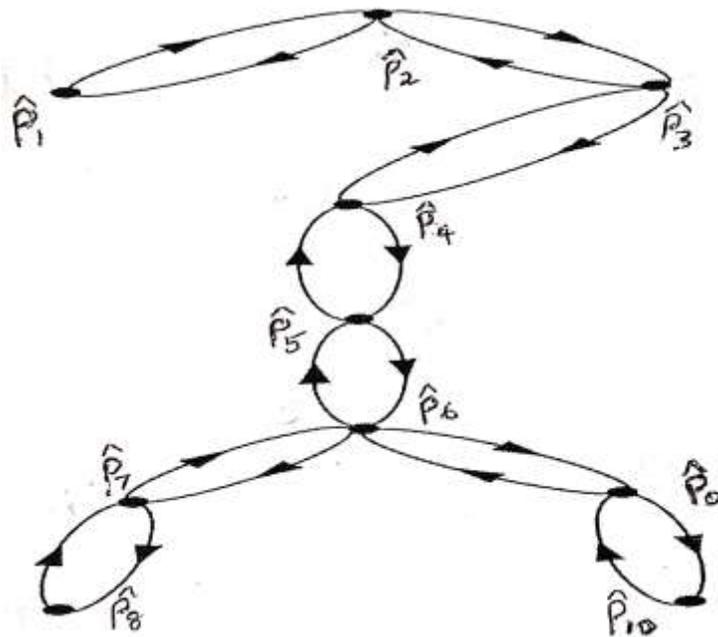
Let  $\hat{G}(\hat{\theta}, \hat{\sigma})$  be a directed-graph (fig.-3) and  $L$  being a sub-graph,  $\hat{\theta}(\hat{G}) = \{\hat{\rho}_1 \text{ to } \hat{\rho}_{10}\}$  be the set of vertices of  $\hat{G}$ , where  $\hat{\rho}_1$  =nostril,  $\hat{\rho}_2$  =nasal cavity,  $\hat{\rho}_3$  =pharynx,  $\hat{\rho}_4$  =larynx,



$\hat{\rho}_5$ =trachea,  $\hat{\rho}_6$ = bronchus,  $\hat{\rho}_7$ =bronchiole (left),  $\hat{\rho}_8$ =bronchiole (right),  $\hat{\rho}_9$ =lung (left),  $\hat{\rho}_{10}$  =lung (right). We establish the foundational initial rightneighborhoods corresponding to every vertex in this work.



**Figure 4.1:**Human lungsmodeling



**Figure 4.2:** Digraph of the Human Lungsmodeling

**Table 4.1:**Initial right neighbourhods of Figure-4.2

Right Neighbourhood	Initial right neighbourhods
$N_r(\hat{\rho}_1) = \{\hat{\rho}_1 \text{ and } \hat{\rho}_2\}$	$N_r^i(\hat{\rho}_1) = \{\hat{\rho}_1 \text{ and } \hat{\rho}_2\}$
$N_r(\hat{\rho}_2) = \{\hat{\rho}_1, to, \hat{\rho}_3\}$	$N_r^i(\hat{\rho}_2) = \{\hat{\rho}_2\}$



$N_r(\hat{\rho}_3) = \{\hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4\}$	$N_r^i(\hat{\rho}_3) = \{\hat{\rho}_3\}$
$N_r(\hat{\rho}_4) = \{\hat{\rho}_3, \hat{\rho}_4, \hat{\rho}_5\}$	$N_r^i(\hat{\rho}_4) = \{\hat{\rho}_4\}$
$N_r(\hat{\rho}_5) = \{\hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_6\}$	$N_r^i(\hat{\rho}_5) = \{\hat{\rho}_5\}$
$N_r(\hat{\rho}_6) = \{\hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_7, \hat{\rho}_9\}$	$N_r^i(\hat{\rho}_6) = \{\hat{\rho}_6\}$
$N_r(\hat{\rho}_7) = \{\hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_8\}$	$N_r^i(\hat{\rho}_7) = \{\hat{\rho}_6, \hat{\rho}_7\}$
$N_r(\hat{\rho}_8) = \{\hat{\rho}_7, \hat{\rho}_8\}$	$N_r^i(\hat{\rho}_8) = \{\hat{\rho}_8\}$
$N_r(\hat{\rho}_9) = \{\hat{\rho}_6, \hat{\rho}_9, \hat{\rho}_{10}\}$	$N_r^i(\hat{\rho}_9) = \{\hat{\rho}_{10}\}$
$N_r(\hat{\rho}_{10}) = \{\hat{\rho}_9, \hat{\rho}_{10}\}$	$N_r^i(\hat{\rho}_{10}) = \{\hat{\rho}_9, \hat{\rho}_{10}\}$

**Table 4.2: Initial right Nano topologies  $\hat{\tau}_{N_r^i}[\hat{\theta}(\hat{L})]$  on subgraphs of Figure-4.2**

$\hat{\theta}(\hat{L})$	$\hat{\tau}_{N_r^i}[\hat{\theta}(\hat{L})]$
$\{\hat{\rho}_1, \hat{\rho}_2\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_2\}\}$
$\{\hat{\rho}_1, \hat{\rho}_4\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_4\}, \{\hat{\rho}_1, \hat{\rho}_4\}\}$
$\{\hat{\rho}_1, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_2, \hat{\rho}_3\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_2, \hat{\rho}_3\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3\}\}$
$\{\hat{\rho}_2, \hat{\rho}_6\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_1, \hat{\rho}_6\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_6\}\}$
$\{\hat{\rho}_3, \hat{\rho}_4\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3, \hat{\rho}_4\}\}$
$\{\hat{\rho}_3, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3\}, \{\hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_3, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_4, \hat{\rho}_8\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_4, \hat{\rho}_8\}\}$
$\{\hat{\rho}_6, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_6\}, \{\hat{\rho}_6, \hat{\rho}_{10}\}, \{\hat{\rho}_6, \hat{\rho}_{10}, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_9, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3\}\}$
$\{\hat{\rho}_1, \hat{\rho}_3, \hat{\rho}_8\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3, \hat{\rho}_8\}, \{\hat{\rho}_1, \hat{\rho}_3, \hat{\rho}_8\}\}$
$\{\hat{\rho}_1, \hat{\rho}_5, \hat{\rho}_8\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_5, \hat{\rho}_8\}, \{\hat{\rho}_1, \hat{\rho}_5, \hat{\rho}_8\}\}$
$\{\hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_7\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_7\}, \{\hat{\rho}_2, \hat{\rho}_3\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_7\}\}$
$\{\hat{\rho}_2, \hat{\rho}_5, \hat{\rho}_8\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_2, \hat{\rho}_5, \hat{\rho}_8\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_5, \hat{\rho}_8\}\}$
$\{\hat{\rho}_3, \hat{\rho}_5, \hat{\rho}_6\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3, \hat{\rho}_5, \hat{\rho}_6\}\}$
$\{\hat{\rho}_3, \hat{\rho}_7, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3\}, \{\hat{\rho}_3, \hat{\rho}_7, \hat{\rho}_{10}\}, \{\hat{\rho}_3, \hat{\rho}_7, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_4, \hat{\rho}_8, \hat{\rho}_9\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_4, \hat{\rho}_8\}, \{\hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_4, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4\}\}$
$\{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_7\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_7\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_7\}\}$





$\{\hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_9, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_3 \text{ to } \hat{\rho}_9\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_3 \text{ to } \hat{\rho}_8\}, \{\hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_7, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1 \text{ to } \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_9\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_1 \text{ to } \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_8\}, \{\hat{\rho}_1 \text{ to } \hat{\rho}_4, \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1, \hat{\rho}_3, \hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_1, \hat{\rho}_3, \hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_8\}, \{\hat{\rho}_1, \hat{\rho}_3 \text{ to } \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_2 \text{ to } \hat{\rho}_5, \hat{\rho}_7, \hat{\rho}_8, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_2 \text{ to } \hat{\rho}_5, \hat{\rho}_7, \hat{\rho}_8\}, \{\hat{\rho}_1 \text{ to } \hat{\rho}_5, \hat{\rho}_7 \text{ to } \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_4 \text{ to } \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_4 \text{ to } \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1 \text{ to } \hat{\rho}_5, \hat{\rho}_7, \hat{\rho}_8\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1 \text{ to } \hat{\rho}_5, \hat{\rho}_7, \hat{\rho}_8\}\}$
$\{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_4, \hat{\rho}_5, \hat{\rho}_6, \hat{\rho}_8, \hat{\rho}_9, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1 \text{ to } \hat{\rho}_9\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_9, \hat{\rho}_{10}\}, \{\hat{\rho}_1 \text{ to } \hat{\rho}_8\}, \{\hat{\rho}_1 \text{ to } \hat{\rho}_8, \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_1, \hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}, \{\hat{\rho}_1, \hat{\rho}_3 \text{ to } \hat{\rho}_{10}\}\}$
$\{\hat{\rho}_2 \text{ to } \hat{\rho}_{10}\}$	$\{\hat{\theta}(\hat{G}), \varphi, \{\hat{\rho}_1\}, \{\hat{\rho}_2 \text{ to } \hat{\rho}_{10}\}, \{\hat{\rho}_1, \hat{\rho}_2 \text{ to } \hat{\rho}_{10}\}\}$

**CONCLUSION**

This study developed an initial right nano topological framework using initial right neighborhood structures derived from directed graphs. A lung based digraph model was used to analyze initial right lower and initial right upper approximations along with initial right boundary regions. The results show that combining initial right nano topology with graph theory provides an effective approach for modeling complex systems. This work offers a foundation for further research in applied nano-topological structures.

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